Please write clearly in block capitals.

Centre number | | | | | | Candidate number | | | | |

Surname ______________________

Forename(s) ______________________

Candidate signature ______________________

A-level
MATHEMATICS
Unit Pure Core 4

Friday 17 June 2016 Afternoon Time allowed: 1 hour 30 minutes

Materials
For this paper you must have:
- the blue AQA booklet of formulae and statistical tables.

You may use a graphics calculator.

Instructions
- Use black ink or black ball-point pen. Pencil should only be used for drawing.
- Fill in the boxes at the top of this page.
- Answer **all** questions.
- Write the question part reference (eg (a), (b)(i) etc) in the left-hand margin.
- You must answer each question in the space provided for that question. If you require extra space, use an AQA supplementary answer book; do **not** use the space provided for a different question.
- Do not write outside the box around each page.
- Show all necessary working; otherwise marks for method may be lost.
- Do all rough work in this book. Cross through any work that you do not want to be marked.

Information
- The marks for questions are shown in brackets.
- The maximum mark for this paper is 75.

Advice
- Unless stated otherwise, you may quote formulae, without proof, from the booklet.
- You do not necessarily need to use all the space provided.

Answer **all** questions.

Answer each question in the space provided for that question.

1 (a) Express $\dfrac{19x-3}{(1+2x)(3-4x)}$ in the form $\dfrac{A}{1+2x}+\dfrac{B}{3-4x}$.

[3 marks]

(b) (i) Find the binomial expansion of $\dfrac{19x-3}{(1+2x)(3-4x)}$ up to and including the term in x^2.

[7 marks]

(ii) State the range of values of x for which this expansion is valid.

[1 mark]

QUESTION PART REFERENCE

Answer space for question 1

QUESTION PART REFERENCE	Answer space for question 1

2 By forming and solving a suitable quadratic equation, find the solutions of the equation

$$3\cos 2\theta - 5\cos\theta + 2 = 0$$

in the interval $0° < \theta < 360°$, giving your answers to the nearest $0.1°$.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 2

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 2

Turn over ▶

3 (a) Express $\dfrac{3 + 13x - 6x^2}{2x - 3}$ in the form $Ax + B + \dfrac{C}{2x - 3}$.

[4 marks]

(b) Show that $\displaystyle\int_3^6 \frac{3 + 13x - 6x^2}{2x - 3}\,\mathrm{d}x = p + q\ln 3$, where p and q are rational numbers.

[4 marks]

QUESTION PART REFERENCE

Answer space for question 3

QUESTION PART REFERENCE	**Answer space for question 3**

Turn over ▶

Do not write outside the box

4 The mass of radioactive atoms in a substance can be modelled by the equation

$$m = m_0 k^t$$

where m_0 grams is the initial mass, m grams is the mass after t days and k is a constant. The value of k differs from one substance to another.

(a) (i) A sample of radioactive iodine reduced in mass from 24 grams to 12 grams in 8 days.

Show that the value of the constant k for this substance is 0.917004, correct to six decimal places.

[1 mark]

(ii) A similar sample of radioactive iodine reduced in mass to 1 gram after 60 days.

Calculate the initial mass of this sample, giving your answer to the nearest gram.

[2 marks]

(b) The half-life of a radioactive substance is the time it takes for a mass of m_0 to reduce to a mass of $\frac{1}{2}m_0$.

A sample of radioactive vanadium reduced in mass from exactly 10 grams to 8.106 grams in 100 days.

Find the half-life of radioactive vanadium, giving your answer to the nearest day.

[4 marks]

QUESTION PART REFERENCE	Answer space for question 4

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 4**

Turn over ▶

5 It is given that $\sin A = \dfrac{\sqrt{5}}{3}$ and $\sin B = \dfrac{1}{\sqrt{5}}$, where the angles A and B are both acute.

(a) (i) Show that the exact value of $\cos B = \dfrac{2}{\sqrt{5}}$.

[1 mark]

(ii) Hence show that the exact value of $\sin 2B$ is $\dfrac{4}{5}$.

[2 marks]

(b) (i) Show that the exact value of $\sin(A - B)$ can be written as $p(5 - \sqrt{5})$, where p is a rational number.

[4 marks]

(ii) Find the exact value of $\cos(A - B)$ in the form $r + s\sqrt{5}$, where r and s are rational numbers.

[3 marks]

QUESTION PART REFERENCE	**Answer space for question 5**

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 5**

Turn over ▶

6 The line l_1 passes through the point $A(0,\ 6,\ 9)$ and the point $B(4,\ -6,\ -11)$.

The line l_2 has equation $\mathbf{r} = \begin{bmatrix} -1 \\ 5 \\ -2 \end{bmatrix} + \lambda \begin{bmatrix} 3 \\ -5 \\ 1 \end{bmatrix}$.

(a) The acute angle between the lines l_1 and l_2 is θ.

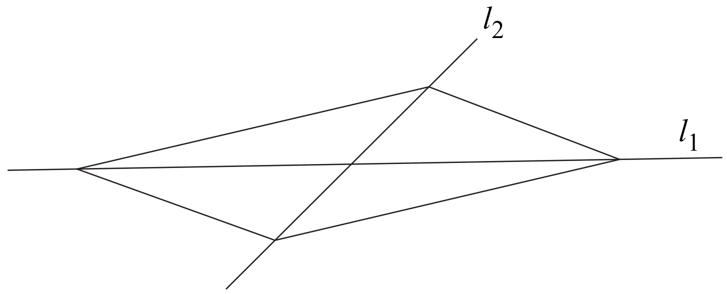
Find the value of $\cos\theta$ as a fraction in its lowest terms.

[5 marks]

(b) Show that the lines l_1 and l_2 intersect and find the coordinates of the point of intersection.

[5 marks]

(c) The points C and D lie on line l_2 such that $ACBD$ is a parallelogram.



The length of AB is three times the length of CD.

Find the coordinates of the points C and D.

[5 marks]

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

QUESTION PART REFERENCE

Answer space for question 6

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 6**

Turn over ▶

7 A curve C is defined by the parametric equations

$$x = \frac{4 - e^{2-6t}}{4}, \qquad y = \frac{e^{3t}}{3t}, \qquad t \neq 0$$

(a) Find the exact value of $\frac{dy}{dx}$ at the point on C where $t = \frac{2}{3}$.

[5 marks]

(b) Show that $x = \frac{4 - e^{2-6t}}{4}$ can be rearranged into the form $e^{3t} = \frac{e}{2\sqrt{(1-x)}}$.

[2 marks]

(c) Hence find the Cartesian equation of C, giving your answer in the form

$$y = \frac{e}{f(x)[1 - \ln(f(x))]}$$

[2 marks]

QUESTION PART REFERENCE

Answer space for question 7

Do not write outside the box

QUESTION PART REFERENCE	**Answer space for question 7**

Turn over ▶

8 It is given that $\theta = \tan^{-1}\left(\dfrac{3x}{2}\right)$.

(a) By writing $\theta = \tan^{-1}\left(\dfrac{3x}{2}\right)$ as $2\tan\theta = 3x$, use implicit differentiation to show that $\dfrac{d\theta}{dx} = \dfrac{k}{4 + 9x^2}$, where k is an integer.

[3 marks]

(b) Hence solve the differential equation

$$9y(4 + 9x^2)\frac{dy}{dx} = \operatorname{cosec} 3y$$

given that $x = 0$ when $y = \dfrac{\pi}{3}$. Give your answer in the form $g(y) = h(x)$.

[7 marks]

QUESTION PART REFERENCE	**Answer space for question 8**

Do not write outside the box

QUESTION PART REFERENCE

Answer space for question 8

Do not write outside the box

QUESTION PART REFERENCE	Answer space for question 8

END OF QUESTIONS