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1. Given that

$$\tan \theta^\circ = p, \text{ where } p \text{ is a constant, } p \neq \pm 1$$

use standard trigonometric identities, to find in terms of p ,

- | | |
|-------------------------------|------------|
| (a) $\tan 2\theta^\circ$ | (2) |
| (b) $\cos \theta^\circ$ | (2) |
| (c) $\cot(\theta - 45)^\circ$ | (2) |

Write each answer in its simplest form.



2. Given that

$$f(x) = 2e^x - 5, \quad x \in \mathbb{R}$$

(a) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$

(ii) $y = |f(x)|$

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(b) Deduce the set of values of x for which $f(x) = |f(x)|$

(1)

(c) Find the exact solutions of the equation $|f(x)| = 2$

(3)



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Question 2 continued



3. $g(\theta) = 4 \cos 2\theta + 2 \sin 2\theta$

Given that $g(\theta) = R \cos(2\theta - \alpha)$, where $R > 0$ and $0 < \alpha < 90^\circ$,

(a) find the exact value of R and the value of α to 2 decimal places. **(3)**

(b) Hence solve, for $-90^\circ < \theta < 90^\circ$,

$$4 \cos 2\theta + 2 \sin 2\theta = 1$$

giving your answers to one decimal place. **(5)**

Given that k is a constant and the equation $g(\theta) = k$ has no solutions,

(c) state the range of possible values of k . **(2)**

Horizontal lines for writing answers.



6.

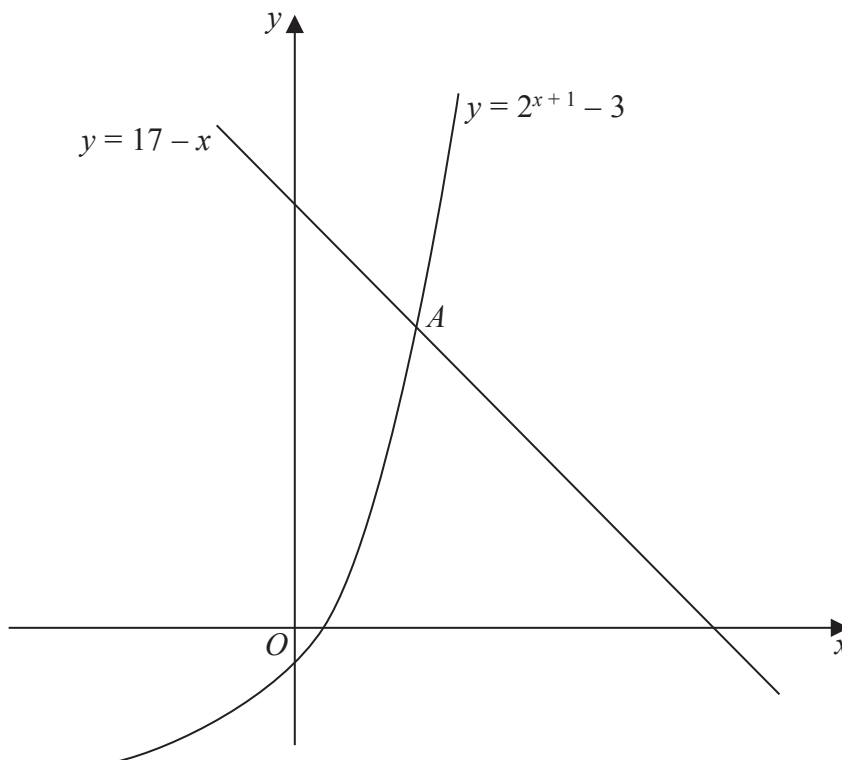


Figure 1

Figure 1 is a sketch showing part of the curve with equation $y = 2^{x+1} - 3$ and part of the line with equation $y = 17 - x$.

The curve and the line intersect at the point A .

(a) Show that the x coordinate of A satisfies the equation

$$x = \frac{\ln(20 - x)}{\ln 2} - 1 \tag{3}$$

(b) Use the iterative formula

$$x_{n+1} = \frac{\ln(20 - x_n)}{\ln 2} - 1, \quad x_0 = 3$$

to calculate the values of x_1, x_2 and x_3 , giving your answers to 3 decimal places. (3)

(c) Use your answer to part (b) to deduce the coordinates of the point A , giving your answers to one decimal place. (2)



7.

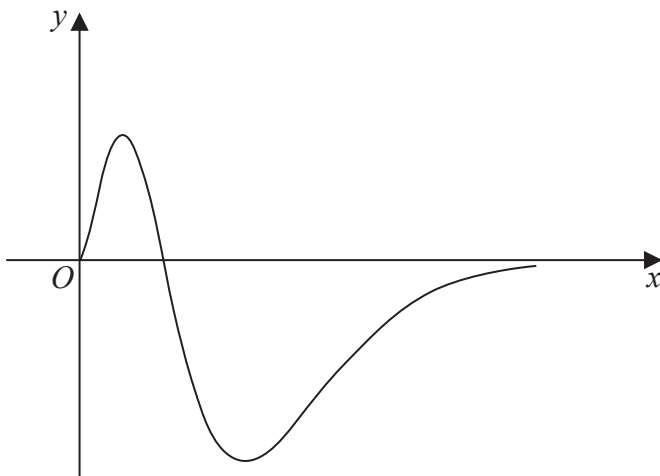


Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \quad x \geq 0$$

- (a) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found. **(3)**

- (b) Hence find the range of g . **(6)**

- (c) State a reason why the function $g^{-1}(x)$ does not exist. **(1)**



8. (a) Prove that

$$\sec 2A + \tan 2A \equiv \frac{\cos A + \sin A}{\cos A - \sin A}, \quad A \neq \frac{(2n + 1)\pi}{4}, \quad n \in \mathbb{Z} \qquad (5)$$

(b) Hence solve, for $0 \leq \theta < 2\pi$,

$$\sec 2\theta + \tan 2\theta = \frac{1}{2}$$

Give your answers to 3 decimal places.

(4)



9. Given that k is a **negative** constant and that the function $f(x)$ is defined by

$$f(x) = 2 - \frac{(x - 5k)(x - k)}{x^2 - 3kx + 2k^2}, \quad x \geq 0$$

(a) show that $f(x) = \frac{x + k}{x - 2k}$ **(3)**

(b) Hence find $f'(x)$, giving your answer in its simplest form. **(3)**

(c) State, with a reason, whether $f(x)$ is an increasing or a decreasing function.
Justify your answer. **(2)**



